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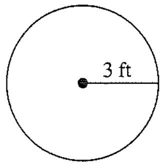
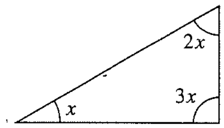
10. *Area and Circumference* Find the area and circumference of the circle shown.



11. *Angles* Find the value of x in the triangle shown.



12. *Solving a Formula* Solve $A = \pi r^2 + \pi rl$ for l.

2.5 LINEAR INEQUALITIES

Solutions and Number Line Graphs ▪ The Addition Property of Inequalities ▪ The Multiplication Property of Inequalities ▪ Applications

A LOOK INTO MATH ▷

At an amusement park, a particular ride might be restricted to people at least 48 inches tall. A child who is x inches tall may go on the ride if $x \geq 48$ but may not go on the ride if $x < 48$. A height of 48 inches represents the boundary between being allowed on the ride and being denied access to the ride. A posted height restriction, or *boundary*, allows parents to easily determine if their child may go on the ride.

Solving linear inequalities is closely related to solving linear equations because equality is the boundary between *greater than* and *less than*. In this section we discuss techniques used to solve linear inequalities.

Solutions and Number Line Graphs

A **linear inequality** results whenever the equals sign in a linear equation is replaced with any one of the symbols $<, \leq, >,$ or $\geq$. Examples of linear equations include

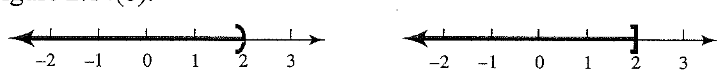
$$x = 5, \quad 2x + 1 = 0, \quad 1 - x = 6, \quad \text{and} \quad 5x + 1 = 3 - 2x.$$

Therefore examples of linear inequalities include

$$x > 5, \quad 2x + 1 < 0, \quad 1 - x \geq 6, \quad \text{and} \quad 5x + 1 \leq 3 - 2x.$$

A **solution** to an inequality is a value of the variable that makes the statement true. The set of all solutions is called the **solution set.** Two inequalities are *equivalent* if they have the same solution set. Inequalities frequently have infinitely many solutions. For example, the solution set for the inequality $x > 5$ includes all real numbers greater than 5.

A number line can be used to graph the solution set for an inequality. The graph of all real numbers satisfying $x < 2$ is shown in Figure 2.14(a), and the graph of all real numbers satisfying $x \leq 2$ is shown in Figure 2.14(b). (The symbol $\leq$ is read "less than or equal to." Similarly, the symbol $\geq$ is read "greater than or equal to.") A parenthesis ")" is used to show that 2 is not included in Figure 2.14(a), and a bracket "]" is used to show that 2 is included in Figure 2.14(b).



(a) $x < 2$ (b) $x \leq 2$

Figure 2.14

EXAMPLE 1 Graphing inequalities on a number line

Use a number line to graph the solution set to each inequality.

(a) $x > 0$ **(b)** $x \geq 0$ **(c)** $x \leq -1$ **(d)** $x < 3$

Solution

(a) First locate $x = 0$ (or the origin) on a number line. Numbers greater than 0 are located to the right of the origin, so shade the number line to the right of the origin. Because $x > 0$, the number 0 is not included, so place a parenthesis "(" at 0, as shown in Figure 2.15(a).

(b) Figure 2.15(b) is similar to the graph in part (a) except that a bracket "[" is placed at the origin because 0 is included in the solution set.

(c) First locate $x = -1$ on the number line. Numbers less than -1 are located to the left of -1. Because -1 is included, a bracket "]" is placed at -1, as shown in Figure 2.15(c).

(d) Real numbers less than 3 are graphed in Figure 2.15(d).

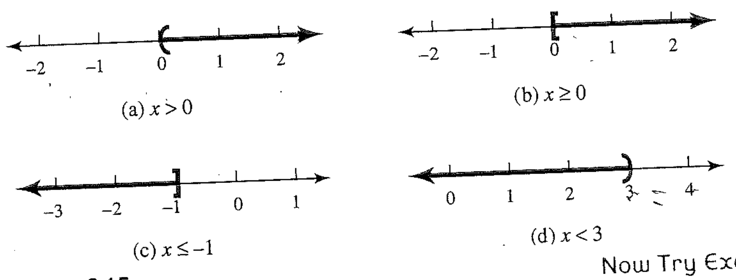


Figure 2.15

Now Try Exercises 13, 17

INTERVAL NOTATION (OPTIONAL) Each number line graph in Figure 2.15 represents an interval of real numbers that corresponds to the solution set to an inequality. These solution sets can also be represented in a convenient notation called **interval notation**. Rather than draw the entire number line, we can use brackets or parentheses to indicate the interval of values that represent the solution set. For example, the number line shown in Figure 2.15(a) can be represented by the interval $(0, \infty)$, and the number line shown in Figure 2.15(b) can be represented by the interval $[0, \infty)$. The symbol ∞ refers to infinity and is used to indicate that the values increase without bound. Similarly, $-\infty$ can be used when the values decrease without bound. The number lines shown in Figures 2.15(c) and (d) can be represented by $(-\infty, -1]$ and $(-\infty, 3)$, respectively.

EXAMPLE 2 Writing solution sets in interval notation

Write the solution set to each inequality in interval notation.

(a) $x > 4$ **(b)** $y \leq -3$ **(c)** $z \geq -1$

Solution

(a) Real numbers greater than 4 are represented by the interval $(4, \infty)$.

(b) Real numbers less than or equal to -3 are represented by the interval $(-\infty, -3]$.

(c) The solution set is represented by the interval $[-1, \infty)$.

Now Try Exercise 27

CHECKING SOLUTIONS We can check possible solutions to an inequality in the same way that we checked possible solutions to an equation. For example, to check whether 5 is a solution to $2x + 3 = 13$, we substitute $x = 5$ in the equation.

$$2(5) + 3 \stackrel{?}{=} 13 \quad \text{Replace } x \text{ with 5.}$$
$$13 = 13 \quad \text{A true statement}$$

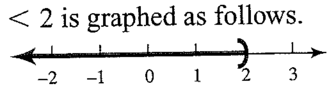
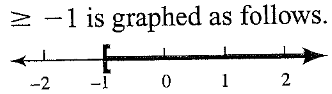
(d) To determine how many laptops need to be sold to yield a positive profit, we must solve the inequality $P > 0$.

$530x - 200{,}000 > 0$	Inequality to be solved
$530x > 200{,}000$	Add 200,000 to each side.
$x > \dfrac{200{,}000}{530}$	Divide each side by 530.

Because $\frac{200{,}000}{530} \approx 377.4$, the company must sell at least 378 laptops. Note that the company cannot sell a fraction of a laptop. Now Try Exercise 119

2.5 PUTTING IT ALL TOGETHER

In this section we discussed linear inequalities and how to solve them. A linear inequality has infinitely many solutions and can be solved by using the addition and multiplication properties of inequalities. When multiplying or dividing an inequality by a negative number, we must reverse the inequality symbol. The following table summarizes some of the concepts presented in this section.

Concept	Comments	Examples
Linear Inequality	If the equals sign in a linear equation is replaced with $<$, $>$, $\le$, or $\ge$, a linear inequality results.	*Linear Equation* / *Linear Inequality* $4x - 1 = 0$ / $4x - 1 > 0$ $2 - x = 3x$ / $2 - x \le 3x$ $4(x + 3) = 1 - x$ / $4(x + 3) < 1 - x$ $-6x + 3 = 5$ / $-6x + 3 \ge 5$
Solution to an Inequality	A value for a variable that makes the inequality a true statement	5 is a solution to $2x > 5$ because $2(5) > 5$ is a true statement.
Set-Builder Notation	A notation that can be used to identify the solution set to an inequality	The solution set for $x - 2 < 5$ can be written as $\{x \mid x < 7\}$ and is read "the set of real numbers x such that x is less than 7."
Solution Set to an Inequality	The set of all solutions to an inequality	The solution set to $x + 1 > 5$ is given by $x > 4$ and can be written in set-builder notation as $\{x \mid x > 4\}$.
Number Line Graphs	The solutions to an inequality can be graphed on a number line.	$x < 2$ is graphed as follows.  $x \ge -1$ is graphed as follows. 

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Concept	Comments	Examples
Addition Property of Inequalities	$a < b$ is equivalent to $a + c < b + c$, where a, b, and c represent real number expressions.	$x - 5 \geq 6$ Given inequality $x \geq 11$ Add 5. $3x > 5 + 2x$ Given inequality $x > 5$ Subtract $2x$.
Multiplication Property of Inequalities	$a < b$ is equivalent to $ac < bc$ when $c > 0$, and is equivalent to $ac > bc$ when $c < 0$.	$\frac{1}{2}x \geq 6$ Given inequality $x \geq 12$ Multiply by 2. $-3x > 5$ Given inequality $x < -\frac{5}{3}$ Divide by -3; reverse the inequality symbol.

2.5 Exercises

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CONCEPTS

1. A linear inequality results whenever the ____ in a linear equation is replaced by any one of the symbols ____, ____, ____, or ____.

2. Equality is the boundary between ____ and ____.

3. A(n) ____ is a value of the variable that makes an inequality statement true.

4. Two linear inequalities are ____ if they have the same solution set.

5. When a linear equation is solved, the solution set contains (one/infinitely many) solution(s).

6. When a linear inequality is solved, the solution set contains (one/infinitely many) solution(s).

7. The solution set to a linear inequality can be graphed by using a ____.

8. The value of 5 (is/is not) a solution to the inequality $3x < 10$.

9. The addition property of inequalities states that if $a > b$, then $a + c$ ____ $b + c$.

10. The multiplication property of inequalities states that if $a < b$ and $c > 0$, then ac ____ bc.

11. The multiplication property of inequalities states that if $a < b$ and $c < 0$, then ac ____ bc.

12. Are $-4x < 8$ and $x < -2$ equivalent inequalities? Explain.

SOLUTIONS AND NUMBER LINE GRAPHS

Exercises 13–20: Use a number line to graph the solution set to the inequality.

13. $x < 0$

14. $x > -2$

15. $x > 1$

16. $x < -\frac{5}{2}$

17. $x \leq 1.5$

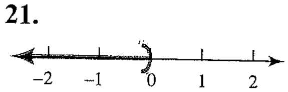
18. $x \geq -3$

19. $z \geq -2$

20. $z \leq -\pi$

Exercises 21–26: Express the set of real numbers graphed on the number line with an inequality.

21.



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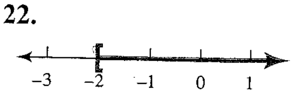

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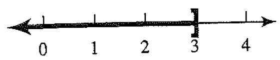

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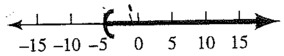

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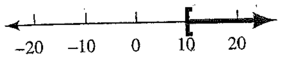

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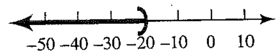

Exercises 27–32: Write the solution set to the inequality in interval notation.

27. $x \geq 6$

28. $x < 3$

29. $y > -2$

30. $y \geq 1$

31. $z \leq 7$

32. $z < -5$

Exercises 33–42: Determine whether the given value of the variable is a solution to the inequality.

33. $x + 5 > 5 \qquad x = 4$

34. $x - 7 < 0 \qquad x = 6$

35. $5x \geq 25 \qquad x = 5$

36. $-3x \leq -8 \qquad x = -2$

37. $4y - 3 \leq 5 \qquad y = -3$

38. $3y + 5 \geq -8 \qquad y = -3$

39. $5(z + 1) < 3z - 7 \qquad z = -7$

40. $-(z + 7) > 3(6 - z) \qquad z = 2$

41. $\frac{3}{2}t - \frac{1}{2} \geq 1 - t \qquad t = -2$

42. $2t - 3 > 5t - (2t + 1) \qquad t = 5$

TABLES AND LINEAR INEQUALITIES

Exercises 43–46: Use the table to solve the inequality.

43. $3x + 6 > 0$

x	−4	−3	−2	−1	0
$3x + 6$	−6	−3	0	3	6

44. $6 - 3x \leq 0$

x	1	2	3	4	5
$6 - 3x$	3	0	−3	−6	−9

45. $-2x + 7 > 5$

x	−1	0	1	2	3
$-2x + 7$	9	7	5	3	1

46. $5(x - 3) \leq 4$

x	3.2	3.4	3.6	3.8	4
$5(x - 3)$	1	2	3	4	5

Exercises 47–50: Complete the table. Then use the table to solve the inequality.

47. $-2x + 6 \leq 0$

x	1	2	3	4	5
$-2x + 6$	4	2	0		−4

48. $3x - 1 < 8$

x	0	1	2	3	4
$3x - 1$	−1				

49. $5 - x > x + 7$

x	−3	−2	−1	0	1
$5 - x$	8				4
$x + 7$	4				8

50. $2(3 - x) \geq -3(x - 2)$

x	−2	−1	0	1	2
$2(3 - x)$					
$-3(x - 2)$					

SOLVING LINEAR INEQUALITIES

Exercises 51–58: Use the addition property of inequalities to solve the inequality. Then graph the solution set.

51. $x - 3 > 0$

52. $x + 6 < 3$

53. $3 - y \leq 5$

54. $8 - y \geq 10$

55. $12 < 4 + z$

56. $2z \leq z + 17$

57. $5 - 2t \geq 10 - t$

58. $-2t > -3t + 1$

Exercises 59–66: Use the multiplication property of inequalities to solve the inequality. Then graph the solution set.

59. $2x < 10$

60. $3x > 9$

61. $-\frac{1}{2}t \geq 1$

62. $-5t \leq -6$

63. $\frac{3}{4} > -5y$

64. $10 \geq -\frac{1}{7}y$

65. $-\frac{2}{3} \leq \frac{1}{7}z$

66. $-\frac{3}{10}z < 11$